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Conversion of Coordinates: Latitude-Longitude to Cislunar

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THE problem of navigation of a space vehicle from Earth to the moon and back again is complicated by the fact that different coordinates are more convenient to use at different phases of the voyage Close to Earth, an Earth-centered latitude-longitude coordinate system or another of the same general type would be most applicable Close to the moon, a moon-centered system would be most applicable In the region between the Earth and the moon, a different set of coordinates might be more applicable such that the X axis is directed along the line of centers of mass of both the Earth and the moon, the Y axis is directed along a line parallel to the plane of the ecliptic, and the Z axis is perpendicular to each of the other axes The origin for this particular coordinate system may be taken as the center of mass of the Earth In further discussions of this particular coordinate system, it will be referred to as the cislunar coordinate system, and a subscript c will be used with the X, Y, Z of this system

It is important that a relation between the various coordinate systems be shown such that a smooth transition between the various coordinate systems can be made As an example, the relationships between an Earth-centered latitude-longitude system of coordinates and the cislunar set of coordinates will be shown with the realization that a transition from the cislunar set of coordinates to an Earth-centered latitude-longitude coordinate system would be made in a similar manner as would transitions involving a moon-centered coordinate system and a cislunar set of coordinates

The Earth-centered latitude-longitude system that will be used will be the one used by Isakson[1] relating a vehicle to inertial space The position of a vehicle is related to inertial space first by a rotation of the coordinate system about a Z axis through a longitude angle Λ and then through a latitude angle λ about the Y axis The X axis will then pass through the centroid of the vehicle A subscript g will be used with quantities referring to this particular coordinate system

In order to relate the position of the moon to the coordinate system involving X, Y, and Z, information regarding the

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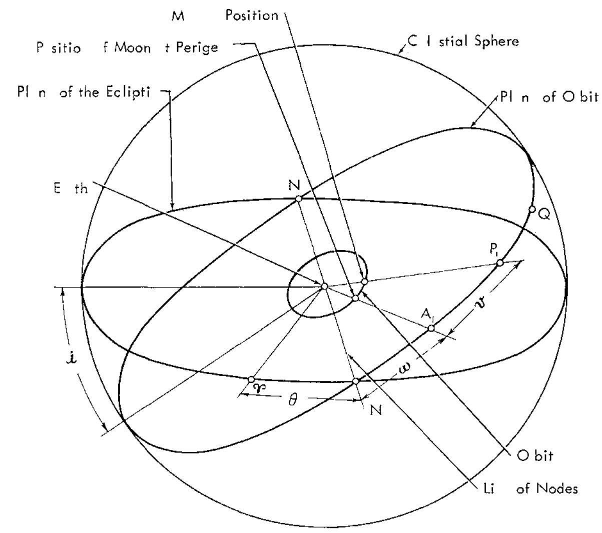

Fig 1 Orbital elements of the moon in its orbit about the Earth

moon's orbit must be known The moon's orbit about the Earth will be inclined at some angle i to the plane of the ecliptic In Fig 1 the plane of the moon's orbit and the plane of the ecliptic intersect in a line N-N' called the line of nodes The angle between the line of nodes and a line extended from the center of Earth through the position of the moon when it is at perigee and measured in the plane of the orbit is denoted as ω In Fig 1 this is indicated by the arc NA_1 The angle in the plane of the ecliptic measured between a line extended from Earth to the first point of Aries and the line of nodes is called the longitude of the ascending node θ In Fig 1 this is indicated by the arc ♈N The longitude of the moon when it is at perigee is defined as $\bar{\omega} = \theta + \omega$ The angular position of the moon at any instant would be given by $\bar{\omega} + v$, where v is the true anomaly In the figure v is increasing; consequently, the arc measured from N refers to the ascending node The true anomaly v would be the angle subtended by a line extending from Earth to perigee of the moon and a line extending from Earth to the position of the moon (arc A_1P_1 of Fig 1)

The eccentricity of the moon's orbit is small; the position of the moon as a function of time will be adequately described by use of mean anomaly rather than true anomaly The mean anomaly is defined by the equation $\bar{v} = \sigma(T - T_0)$, where T_0 is the time that the moon is at perigee and T is the time under consideration such that $(T - T_0)$ is the time that it takes the moon to go from perigee position to its present position In Fig 1 the mean anomaly would correspond to the arc A_1Q Actually, the arcs A_1P_1 and A_1Q are very nearly equal Use of mean anomaly rather than true anomaly simplifies the required calculations considerably with little degradation in accuracy

Having defined the orbital elements, it is now necessary to define the navigational elements The celestial equator is the great circle formed on the celestial sphere by extension of the plane of the equator of the Earth Declination, δ, is measured northward or southward from the celestial equator The sideral hour angle, SHA, is measured westward from the hour circle of the vernal equinox, the first point of Aries (an hour circle being a great circle related to a point on the celestial sphere) The right ascension, RA, is measured eastward from the vernal equinox and is such that RA = 24^h − SHA Figure 2 shows the relationships of these angles as well as their directional sense The Greenwich hour angle, GHA, and local hour angle, LHA, are measured westward from 0° to 360° with the Greenwich or local celestial meridian, respectively, used as a reference Because of the apparent daily rotation of the celestial sphere, the hour

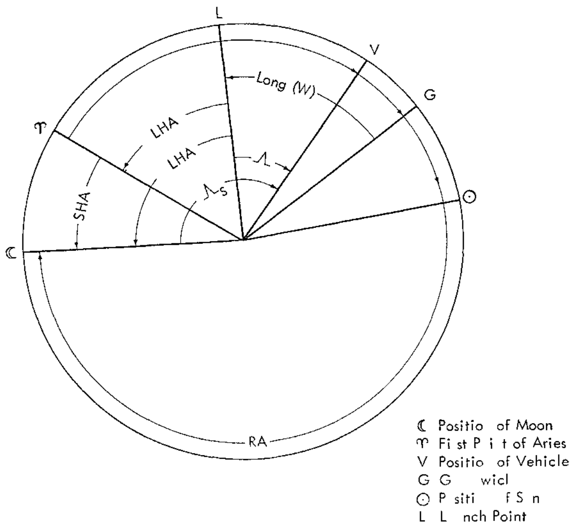

Fig 2 Equatorial plane as viewed from the South Pole

angles continually increase As the celestial sphere rotates, each body crosses any particular celestial meridian approximately once a day

Consulting the time diagram or Fig 2 in order to cause the line through V, which would be a line through the space vehicle's position, to coincide with a line through the position of the moon, a rotation about an axis passing through the poles of the Earth should be made This angle Λ can be expressed as

$$\Lambda = \text{SHA}☾ + \text{LHA}♈ + \Lambda \tag{1}$$

or as

$$\Lambda = \text{LHA}☾ + \Lambda \tag{2}$$

The symbol Λ would be the difference in longitude of the vehicle at launch and the position at some other time The LHA☾ may be made a function of time in terms of some initial LHA_0☾ by considering the equation

$$\text{LHA}☾ = \text{LHA}_0☾ + (\Omega_E - \sigma)(T - T_0) \tag{3}$$

where Ω_E would be the Earth's rotational rate about its own axis, σ would be the rotational rate of the moon about the Earth, t_0 would be the time at which LHA_0☾ was considered, and T would be the time at which LHA would be considered

By modifying some formulas of spherical astronomy, the right ascension and declination of the moon can be found in terms of the orbital elements of the moon with respect to the Earth Smart[2] states that the formulas that have been derived for the motion of a planet about the sun are applicable to the motion of the moon about the Earth Using the formulas as derived by Smart[2] and modifying them, the RA and δ may be obtained such that

$$\text{RA} = \tan^{-1}\left[\frac{\sin b \sin(B + \omega + \bar{v})}{\sin a \sin(A + \omega + \bar{v})}\right] \tag{4}$$

$$\delta = \tan^{-1}\left[\frac{\sin c \sin(C + \omega + \bar{v}) \sin\text{RA}}{\sin b \sin(B + \omega + \bar{v})}\right] \tag{5}$$

where the mean anomaly has been used rather than the true anomaly The auxiliary angles a, b, c, A, B, C are defined by Smart[2] as

$$\sin a \sin A = \cos\theta \tag{6}$$

$$\sin a \cos A = -\sin\theta \cos i \tag{7}$$

$$\sin b \sin B = \sin\theta \cos\epsilon \tag{8}$$

$$\sin b \cos B = \cos\theta \cos i \cos\epsilon - \sin i \sin\epsilon \tag{9}$$

$$\sin c \sin C = \sin\theta \sin\epsilon \tag{10}$$

$$\sin c \cos C = \cos\theta \cos i \sin\epsilon + \sin i \cos\epsilon \tag{11}$$

The angle ϵ is the angle between the plane of the ecliptic and the equatorial plane

The position of the vehicle described in the set of axes with coordinates X , Y , Z can be expressed in terms of the set of axes with coordinates X_g, Y_g, Z_g Unit vectors will be defined such that $\mathbf{X}_g = X_g\mathbf{l}_{xg}$, $\mathbf{Y}_g = Y_g\mathbf{l}_{yg}$, etc , where $\mathbf{l}_{xg}$, $\mathbf{l}_{yg}$ are unit vectors First rotate about the $\mathbf{l}_{yg}$ vector to some intermediate orientation such that the coordinates are X', Y', Z':

$$\begin{Bmatrix} l_x' \\ l_y' \\ l_z' \end{Bmatrix} = [\lambda]^{-1}\begin{Bmatrix} l_{xg} \\ l_{yg} \\ l_{\ g} \end{Bmatrix} \text{ where } [\lambda]^{-1} = \begin{bmatrix} \cos\lambda & 0 & -\sin\lambda \\ 0 & 1 & 0 \\ \sin\lambda & 0 & \cos\lambda \end{bmatrix} \tag{12}$$

There is then a rotation such that l_x' is coplanar with l_x :

$$\begin{Bmatrix} l_x'' \\ l_c'' \\ l\ '' \end{Bmatrix} = [\Lambda_s]^{-1}\begin{Bmatrix} l_x' \\ l_y' \\ l\ ' \end{Bmatrix} \text{ where } [\Lambda\ ]^{-1} = \begin{bmatrix} \cos\Lambda_s & -\sin\Lambda & 0 \\ \sin\Lambda & \cos\Lambda_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The final rotation would be through the angle of declination:

$$\begin{Bmatrix} l_{xc} \\ l_y \\ l_z \end{Bmatrix} = [\delta] \begin{Bmatrix} l_x'' \\ l_y'' \\ l\ '' \end{Bmatrix} \text{ where } [\delta] = \begin{bmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{bmatrix} \tag{14}$$

Consequently, the transformation from the geocentric latitude-longitude frame of reference to the system of coordinates with coordinates X_c, Y , Z_c by means of the following transformation is

$$\begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix} = [\lambda]^{-1}[\Lambda_s]^{-1}[\delta] \begin{Bmatrix} l_{xg} \\ l_{yg} \\ l_{\ g} \end{Bmatrix} \tag{15}$$

In the transformation from the set of axes with coordinates X_g, Y_g, Z_g to the set of axes with coordinates X , Y , Z , two important factors must be considered: the position in one set of coordinates must be related to the position in the other set of axes, and forces related to one set of axes must be related to the other set of axes Using Eq (15) and remembering that $\mathbf{r} = r\mathbf{l}_{xg}$, the following relationships are obtained:

$$X = r(\cos\lambda \cos\Lambda_s \cos\delta + \sin\lambda \sin\delta) \tag{16}$$

$$Y_c = r(\sin\Lambda_s \cos\lambda) \tag{17}$$

$$Z = r(\cos\delta \sin\lambda - \sin\delta \cos\Lambda_s \cos\lambda) \tag{18}$$

If a_{xg}, a_{yg}, $a_{\ g}$ are defined as accelerations in the geocentric latitude-longitude system of coordinates such that $\mathbf{a} = \mathbf{F}/m = a_{xg}\mathbf{l}_{xg} + a_{yg}\mathbf{l}_{yg} + a_{\ g}\mathbf{l}_{\ g}$, these may be related to the cislunar set of coordinates by using Eq (15) such that

$$a_{xc} = a_{xg}(\cos\delta \cos\Lambda_s \cos\lambda + \sin\delta \sin\lambda) - a_{yg} \cos\delta \sin\Lambda_s + a_{zg}(\sin\delta \cos\lambda - \cos\delta \cos\Lambda_s \sin\lambda) \tag{19}$$

$$a_y = a_{xg} \sin\Lambda \cos\lambda + a_{yg} \cos\Lambda - a_{zg} \sin\Lambda \sin\lambda \tag{20}$$

$$a\ = a_{xg}(\cos\delta \sin\lambda - \sin\delta \cos\Lambda_s \cos\lambda) + a_{yg} \sin\delta \sin\Lambda_s + a_{zg}(\cos\delta \cos\lambda + \sin\delta \cos\Lambda \sin\lambda) \tag{21}$$

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Long-Term Coupling Effects between Librational and Orbital Motions of a Satellite

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Introduction

FOR a dynamically nonspherical satellite, Newton's equations of orbital motion are coupled to Euler's equations of rigid-body rotational motion by high-order body-force terms The body force is produced by the earth's gravity gradient and by the nonspherical inertia ellipsoid of the satellite The body force is much smaller than the central force that acts on the satellite as though the satellite is a point mass Their ratio is proportional to L^2/r^2, where L is a characteristic dimension of the satellite and r the geocentric distance of the center of mass of the satellite It is through this small body force that the orbital and the rotational motions affect each other Ordway[1] and Moran[2] have investigated the coupling effects for the case of an undamped dumbbell satellite in a circular orbit, showing that the rotational motion will produce sinusoidal perturbations to the basic orbit In this note, however, a study is presented of the long-term coupling effects for a satellite of general shape, provided with damping, and in an elliptic orbit

In the passive attitude stabilization of a satellite either by gravitational or by magnetic means, it is necessary that the rotational motion of the satellite be provided with damping by internal or external means Damping by external means, as often used in magnetic orientation,[3] is obtained from interaction with, e g, the geomagnetic field by a device provided in the satellite Damping by internal means is produced by the relative motion between the satellite and an auxiliary body contained in or connected to the satellite, and the damping can be either of the viscous type or of the hysteresis type[4] A question is often raised as to where the energy, which is being continuously dissipated through the damping mechanism, comes from One can surmise that the dissipated energy originates from the initial angular momentum about the center of mass of the satellite when entering the orbit, from the sporadic and the continuous momentum inputs from space environments, and from the orbital motion The initial and the subsequent angular momentum inputs will be dissipated into heat through the damping mechanism within a reasonably short time[4] If, however, the rotational motion is excited continuously by the orbital motion (i e, by the orbital eccentricity in the case of gravitational orientation or even by a circular orbital motion in the case of magnetic orientation), then damping of the rotational motion will result in a continuous dissipation of the orbital energy

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In this note we will investigate the nature of the long-term coupling effects between the orbital and the rotational motion in the case of gravitational orientation provided with damping by internal means For this reason it is irrelevant to consider at the same time other effects due to, e g, space environment, earth's oblateness, etc, despite the fact that these latter effects are most likely bigger in magnitude than those due to the rotational motion For the same reason, the present work is restricted to the special case of planar pitch librational motion

Equations for Planar Pitch Motion

It is assumed that earth is a uniform sphere of radius R_0 and that the satellite is a composite body of a total mass m, consisting of a main rigid body and an auxiliary rigid body, connected to each other through a hinge joint such that their centers of mass coincide The coordinate systems are chosen such that the orbit lies in the XZ plane of a nonrotating coordinate system O-XYZ with the orbital angular momentum vector pointing in the Y direction The origin O is situated at the geocenter The mass center o of the satellite is specified by polar coordinates (r,Θ), where Θ is the angle measured from the Z axis in the direction of the orbital motion A rotating coordinate system o-$x'y'z'$ is taken in such a way that oz' is parallel to Oo and oy' to OY The body coordinate systems are chosen with the o-$x_1y_1z_1$ and o-$x_2y_2z_2$ axes of body 1 and 2, respectively, along the principal axes with moments of inertia (I_{xi},I_{yi},I_{zi}), $i = 1,2$ The satellite is assumed to undergo only a pitch rotational motion such that its z_1 and z_2 axes make an angle θ_1 and θ_2, respectively, with the z' axis in the orbital plane, and the y_1 and y_2 axes are always parallel to oy' or OY In the hinge joint of the composite satellite, the spring torque is assumed to be linear with the relative angular displacement (k = spring constant) and the viscous damping torque to be linear with the relative angular velocity between the two bodies (c = damping coefficient) With terms of order higher than L^2/r^2 neglected, the following four equations result:

$$\ddot{r} - r\dot{\Theta}^2 = -\frac{\mu}{r^2} - \frac{3}{2}\frac{\mu}{mr^4}[-3I_{x_1}\sin^2\theta_1 - 3I_{z_1}\cos^2\theta_1 + (I_{x_1} + I_{y_1} + I_{z_1}) - 3I_{x_2}\sin^2\theta_2 - 3I_{z_2}\cos^2\theta_2 + (I_{x_2} + I_{y_2} + I_{z_2})] \quad (1)$$

$$2\dot{r}\dot{\Theta} + r\ddot{\Theta} = 3(\mu/mr^4)[(I_{x_1} - I_{z_1})\sin\theta_1\cos\theta_1 + (I_{x_2} - I_{z_2})\sin\theta_2\cos\theta_2] \quad (2)$$

$$I_{y_1}(\ddot{\Theta} + \ddot{\theta}_1) = -3(\mu/r^3)(I_{x_1} - I_{z_1})\sin\theta_1\cos\theta_1 - c(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2) \quad (3)$$

$$I_{y_2}(\ddot{\Theta} + \ddot{\theta}_2) = -3(\mu/r^3)(I_{x_2} - I_{z_2})\sin\theta_2\cos\theta_2 - c(\dot{\theta}_2 - \dot{\theta}_1) - k(\theta_2 - \theta_1) \quad (4)$$

where dots denote time derivatives and $\mu = gR_0^2$, with g being the gravitational acceleration at the earth's surface

Method of Successive Approximations

Equations (1–4) will be solved by a method of successive approximations First, the body-force terms of $O(L^2/r^2)$ are neglected in (1) and (2) The orbital equations become decoupled from the rotational equations For chosen initial conditions, the Keplerian orbit can be immediately obtained as an ellipse of semimajor axis r_0, phase angle φ_0, and eccentricity $e < 1$ described by the following two relations: $r = r_0(1 - e^2)/[1 + e\cos(\Theta + \varphi_0)]$ and $\dot{\Theta} = [\mu r_0(1 - e^2)]^{1/2}/r^2$ With the aid of the foregoing relations, (3) and (4) can be solved in the librational case, the solutions of which are then in turn substituted into the orbital equations to solve for the first-order solution, or solution including terms of the same order as the body force or $O(L^2/r^2)$